

Joint probabilistic data association filter for partially unresolved target groups

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Abstract – *In many surveillance problems the observed objects are so closely spaced that they cannot always be resolved by the sensor(s). Typical examples for partially unresolved measurements are the surveillance of aircraft in formation, and convoy tracking for ground surveillance. Ignoring the limited sensor resolution in a tracking system may lead to degraded tracking performance, in particular unwanted track-losses. In this paper, we further discuss a recently presented extension of the resolution model by Koch and van Keuk to the case of arbitrary object numbers, and it is shown how that model can be incorporated into the Joint Probabilistic Data Association Filter (JPDAF). Further, through simulations of a ground target tracking scenario, it is shown how the incorporation of the resolution model improves tracking performance when targets are partially unresolved.*

Keywords: Sensor resolution, target tracking, closely spaced targets, group targets, extended objects, negative information.

1 Introduction

Traditionally, limited sensor resolution has not been considered when designing target tracking algorithms. Instead, it has often been assumed that all targets are always resolved [1]. In many situations, this is a reasonable assumption, but there are important cases when the resolution capabilities of the sensor cannot be ignored [2]. Typical examples of when objects are closely spaced in relation to the sensor resolution is the tracking of aircraft in formation, and convoy tracking for ground surveillance. For such applications, ignoring the limited sensor resolution may lead to degraded tracking performance, in particular due to premature deletion of tracks.

The ability of a sensor to resolve several objects can be described by the resolution probability. The corresponding resolution model should approximately cover the essential aspects of the real sensor resolution and should also be mathematically and numerically tractable. An important aspect of sensor resolution is its dependence on the sensor–target

distance. This property has been covered in the resolution model by Koch et al. [3, 4] but is absent in traditional grid-based approaches [5, 6]. Until recently, the existing resolution models have been limited to only two targets, whereas, to the best of our knowledge, a general model for arbitrary target numbers has not existed.

However, in [7] we extended the resolution model in [3], given for two partially unresolved targets, to the case of arbitrary target numbers. A possible integration into the Joint Probabilistic Data Association Filter (JPDAF) [8] was sketched there, too. In this paper, we revisit the proposed resolution model, and we show in more detail how the JPDA filter incorporation can be made. We also present simulation results of a ground target tracking scenario with resolution limitations, where the JPDA filter with the proposed resolution model is shown to yield better tracking results than the ordinary JPDAF. The improved tracking performance is due to a better description of the measurements, which leads to a lower probability of track merging and, hence, smaller risk of premature track deletion.

2 Problem formulation

The problem considered in this paper is how to model the received measurements from an unresolved group of targets, and how that model can be incorporated into a general tracking framework. The ultimate goal of a tracking filter is to calculate the posterior density $p(\mathbf{x}_k | \mathbf{Z}^k)$ of the joint target state vector \mathbf{x}_k , given all measurements \mathbf{Z}^k up to, and including, the current time index k . For the N -target case, the joint target states is described as

$$\mathbf{x}_k = \left[(\mathbf{x}_k^{(1)})^T \quad (\mathbf{x}_k^{(2)})^T \quad \dots \quad (\mathbf{x}_k^{(N)})^T \right]^T, \quad (1)$$

where $\mathbf{x}_k^{(i)}$ is the state of target i . Further, the collection of measurements up to time index k is given by

$$\mathbf{Z}^k = \{ \mathbf{Z}_1, \mathbf{Z}_2, \dots, \mathbf{Z}_k \}. \quad (2)$$

In order to perform the calculation of the posterior density in the presence of resolution uncertainty, we need to

model the measurements from an unresolved group of targets (henceforth called a group target), and to describe the resolution probabilities for that group. We also need to model the motion of the targets.

2.1 Sensor data description

At time index k , a sensor produces observations which are either target-generated or spurious. The observations are gathered in a measurement vector

$$\mathbf{Z}_k = \left[(\mathbf{z}_k^{(1)})^T \quad (\mathbf{z}_k^{(2)})^T \quad \dots \quad (\mathbf{z}_k^{(M_k)})^T \right]^T \quad (3)$$

of length M_k . In this vector, M_t observations are target-originated and M_c are clutter or spurious measurements due to false alarms (henceforth collectively referred to as clutter), but their respective order is unknown. The uncertainty in heritage of the data is described as

$$\mathbf{Z}_k = (\mathbf{\Pi}_p \otimes \mathbf{I}_{N_z \times N_z}) \begin{bmatrix} \mathbf{Z}_k^c \\ \mathbf{Z}_k^t \end{bmatrix}, \quad (4)$$

where \mathbf{Z}_k^c and \mathbf{Z}_k^t are ordered vectors of clutter- and target-originated measurements, respectively, N_z is the dimension of the measurements, \otimes denotes the Kronecker product, and $\mathbf{\Pi}_p$ is an M_k -dimensional random permutation matrix. $\mathbf{\Pi}_p$ models one part of the data association uncertainty, viz. the discrimination between target-generated measurements and clutter measurements, and in which order they appear in the measurement vector \mathbf{Z}_k .

As is common in most tracking algorithms, we assume that the clutter measurements \mathbf{Z}_k^c are described by a spatially homogeneous Poisson process with intensity λ .

To describe the target-originated measurements \mathbf{Z}_k^t under perfect sensor resolution, we need to connect each measurement with a single target, which measurement originated from which target, which is the second part of the data association problem. However, when limited sensor resolution is taken into account, we instead need to consider associating one measurement to a group target (possibly of size one).¹

$$\mathbf{Z}_k^t = (\mathbf{C}_k \otimes \mathbf{I}_{N_z \times N_z}) \tilde{\mathbf{H}} \mathbf{x}_k + \mathbf{w}_k, \quad (5)$$

where \mathbf{C}_k is an unknown M_t -dimensional square matrix which determines the size of the contribution of each target to each measurement. In the classical problem with perfect resolution, \mathbf{C}_k can at most have a single non-zero element (a one) per row. Here, although the sum over each row must still be one, there can be several non-zero elements per row, describing the merged measurement of an unresolved group of targets. The multi-target observation matrix is defined as

$$\tilde{\mathbf{H}} = \text{diag}\{\underbrace{[\mathbf{H}, \dots, \mathbf{H}]}_{N_t \text{ times}}\}, \quad (6)$$

¹We here assume that a merged measurement from an unresolved group can be written as a weighted sum of the positions of the individual targets. In practice, this is not always true, for example due to radar target glint.

where \mathbf{H} is the single-target observation matrix, which describes the relation between target state space and the measurement space². The noise process \mathbf{w}_k is assumed Gaussian with zero mean and block-diagonal covariance matrix \mathbf{R}_k whose blocks depend on if the measurement is from a single target or from a group of targets; see Equation 7. Although the measurement model is assumed linear and Gaussian in this paper, the proposed resolution model does not hinge on linear-Gaussian models, but the description of the exact posterior density is only given for the linear-Gaussian case.

2.2 Group measurement model

In this paper, we assume a simple model for the measurement of an unresolved target group (henceforth referred to as a group measurement). This simple model does not capture the true nature of a group measurement, but serves the purpose of illustrating the proposed framework. In general, any group measurement model can be used within the framework.

The assumed model states that a group measurement can be described as a measurement of the group center, where the center point is given by the arithmetic mean of the involved target states. That is, for an unresolved group of n_g targets (possibly one), whose state vectors are gathered in the joint vector \mathbf{x}_k^g , their group measurement $\mathbf{z}_k^{t,(j)}$ is described by

$$\mathbf{z}_k^{t,(j)} = \mathbf{H}_{n_g} \mathbf{x}_k^g + \mathbf{u}_k^{g,n_g}, \quad (7)$$

where

$$\mathbf{H}_{n_g} = \frac{1}{n_g} \underbrace{[\mathbf{H}, \dots, \mathbf{H}]}_{n_g \text{ times}}, \quad (8)$$

and where $\mathbf{u}_k^{g,n_g} \sim \mathcal{N}(\mathbf{0}, \mathbf{R}_k^{n_g})$ is the measurement noise for an n_g -target group. Typically, the measurement noise covariance matrix increases with increased number of targets in the group.

2.3 Process model

For the prediction of future states of the targets, a process, or motion, model is used. We assume that the motion model is linear and Gaussian, i.e.,

$$\mathbf{x}_{k+1} = \mathbf{F}_k \mathbf{x}_k + \mathbf{v}_k, \quad (9)$$

where \mathbf{F}_k is a known system matrix, and where $\mathbf{v} \sim \mathcal{N}(\mathbf{0}, \mathbf{Q})$ is the Gaussian, zero-mean process noise with covariance \mathbf{Q} .

3 Resolution model for arbitrary target numbers

In this section we revisit the recently proposed resolution model of [7], which we utilize for incorporation in the JPDA

²It is assumed that the relation between the target state space and the measurement state space can be described using a linear mapping.

filter in Section 5. The model describes the resolution probability for a group of arbitrary, but known, number of targets, and it is a generalization of the two-target model in [3].

To describe the event that a group target of arbitrary size is unresolved, and to express the probability of that event, it is necessary to have a simple, yet intuitive, model for the resolution event. The approach of [7] is to have a graph description of a resolution situation, where each node (vertex) in the graph represents a target. Two targets are then mutually unresolved if they are connected by an edge in the graph. Further, a group of targets is unresolved if there exists a walk in the graph through all target vertices. Consequently, there are several graphs which generate the same target group. In Fig. 1, a possible graph for a three-target case is illustrated. The graph is the only graph which represents the event that target one is unresolved while targets two and three form an unresolved group.



Figure 1: Illustration of a graph which describes the event that target one is resolved, while targets two and three form an unresolved group.

If there are more than two targets in a group, there are several graphs which describe that resolution event. In Fig. 2, all possible graphs which represent the unresolvability of a three-target group are illustrated.

What we seek is a probabilistic description of the sensor resolution, where, conditioned on the states of the targets, all possible groups of unresolved targets are described and the probability for each configuration is determined.

3.1 Resolution probabilities

A certain graph describes which targets are connected and which are not, i.e., which targets are pairwise unresolved, and which are pairwise resolved. In the proposed model, for a group to be declared unresolved, not all of the targets in the group have to be pairwise unresolved, but there needs to be a connection between all targets (a walk through the vertices of those targets in the graph). Mathematically spoken, an unresolved group is represented by a simple, not necessarily complete, graph. Consider the illustration in Fig. 2, which shows the four graphs that describe the event that a group of three targets is unresolved. In the leftmost graph of the figure, the edge between target 1 and target 2 resembles the event that these two targets are pairwise unresolved. The edge between target 1 and target 3 states that that pair is unresolved. Since there is a connection between target 2 and target 3 (through target 1), the group of targets $\{1, 2, 3\}$ is unresolved and produces a single group measurement in the model that we propose.

Depending on the geometry of the resolution problem, not all connections between targets are possible. For example,

if we track a convoy of targets which move along a straight road, it is not reasonable that the front and end targets are mutually unresolved, while targets in the middle of the convoy are resolved. In general, for an arbitrary graph, it is sufficient to consider the closest neighboring vertices in each measurement dimension. For the feasible graphs, the resolution model of [7] assumes that the probabilities of resolution are *pairwise independent*. The resolution probability is then the product of all pairwise events. For each pair of targets, the resolution probability is assumed to be captured by the model in [3].

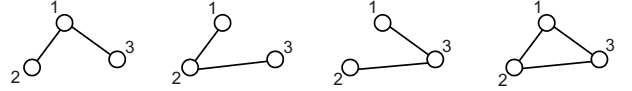


Figure 2: Illustration of the four possible graphs which describe the event that a group of three targets is unresolved.

Consider a general graph, \mathcal{V} , for example as illustrated by one of the graphs in Fig. 2. The probability for that graph, given the target states, is according to the N -target resolution model given by

$$\Pr\{\mathcal{V}|\mathbf{x}_k\} = \prod_{S_e} P_u(S_e) \prod_{S_0} (1 - P_u(S_0)), \quad (10)$$

where S_e is the set of pairwise targets that are connected with an edge ($\{1, 2\}$ and $\{1, 3\}$ in the leftmost graph in Fig. 2), and S_0 is the set of pairwise targets that are not connected ($\{2, 3\}$). Note that S_0 and S_e are given by \mathcal{V} , and P_u is a function of the multi-target state \mathbf{x}_k . Further, we use the definition of the product over an empty set: $\prod_{i \in \emptyset} C \triangleq 1$.

3.1.1 Two-target resolution model

For a certain pair of targets $\mathbf{x}_k^{(i)}$, $\mathbf{x}_k^{(j)}$ in a set \mathcal{S} in (10), the probability that they are unresolved is, according to [3],

$$P_u(\mathbf{x}_k^{(i)}, \mathbf{x}_k^{(j)}) = \prod_{l=1}^{N_{\text{res}}} e^{-\ln(2) \left(\frac{\Delta^{i,j} x_l}{\alpha_{x_l}} \right)^2}, \quad (11)$$

where N_{res} is the dimension of the measurement space (2 for range and azimuth), $\Delta^{i,j} x_l$ is the distance between the predicted positions of targets i and j in dimension x_l , and α_{x_l} describes the resolution capabilities of the sensor in dimension x_l . The dimension can for example be $x_l = r$ for range, or $x_l = \varphi$ for azimuth angle. The probability $P_u(\mathbf{x}_k^{(i)}, \mathbf{x}_k^{(j)})$ can also be written as a scaled multivariate Gaussian [3]

$$P_u(\mathbf{x}_k^{(i)}, \mathbf{x}_k^{(j)}) = |2\pi \mathbf{R}_{u, N_{\text{res}}}|^{1/2} \mathcal{N}(0; \Delta^{i,j} \mathbf{x}_k, \mathbf{R}_{u, N_{\text{res}}}), \quad (12)$$

where $\Delta^{i,j} \mathbf{x}_k$ is a vector with the distances between the target positions in each dimension, and where

$$\mathbf{R}_{u, N_{\text{res}}} = \frac{1}{(2 \ln(2))^{N_{\text{res}}/2}} \text{diag}\{\alpha_{x_1}^2, \dots, \alpha_{x_{N_{\text{res}}}}^2\}. \quad (13)$$

For range and azimuth measurements, and small $\Delta\varphi$, this can easily be generalized to Cartesian coordinates by a rotation by the average angle $\bar{\varphi} = (\varphi^{(1)} + \varphi^{(2)})/2$.

3.2 Graph likelihood

In this section we derive the likelihood $p(\mathbf{Z}_k|\mathcal{V}, \mathbf{x}_k)$ of a graph and the joint state vector, given the observed data at the current time index. The likelihood function cannot be described without conditioning on the association of data to the single targets and group targets in the graph \mathcal{V} . That is, we cannot express the likelihood without knowing the permutation matrix $\mathbf{\Pi}_p$ in (4) and the matrix \mathbf{C}_k in (5). In this section, we assume that the data heritage is known, i.e., that we know these matrices. The information about the data associations is gathered in the association vector d , whose j^{th} element is $i > 0$ if measurement j is assigned to target group i , and $i = 0$ if the measurement is due to clutter. The length of the d vector is equal to the total number of measurements, M_k .

Conditioned on the data association, it is known which of the measurements in \mathbf{Z}_k are target-generated and which originate from clutter. That is, the vectors \mathbf{Z}_k^c (containing M_c observations) and \mathbf{Z}_k^t (containing M_t observations) are known, and we can express their densities as $p(\mathbf{Z}_k^c|\mathcal{V}, d, \mathbf{x}_k)$ and $p(\mathbf{Z}_k^t|\mathcal{V}, d, \mathbf{x}_k)$. We start with the clutter density.

As described in the problem formulation, the clutter observations are assumed spatially uniform in the measurement space. The clutter density is hence

$$p(\mathbf{Z}_k^c|\mathcal{V}, d, \mathbf{x}_k) = \frac{1}{|\text{FoV}|^{M_c}}, \quad (14)$$

where FoV is the field of view of the sensor.

Knowing the data association vector d and the graph \mathcal{V} , the density of target-generated measurements can be described as

$$p(\mathbf{Z}_k^t|\mathcal{V}, d, \mathbf{x}_k) = \prod_{i=1}^{M_t} p(\mathbf{z}_k^{t,(i)}|d, \mathcal{V}, \mathbf{x}_k), \quad (15)$$

where $p(\mathbf{z}_k^{t,(i)}|d, \mathcal{V}, \mathbf{x}_k)$ is given by the group measurement model in (7).

Finally, the likelihood of a graph and the joint state vector, given the observed data at the current time is

$$p(\mathbf{Z}_k|\mathcal{V}, d, \mathbf{x}_k) = \frac{1}{|\text{FoV}|^{M_c}} \prod_{i=1}^{M_t} p(\mathbf{z}_k^{t,(i)}|d, \mathcal{V}, \mathbf{x}_k). \quad (16)$$

4 Calculation of the posterior density function

In this section, we describe the calculation of the posterior probability density function (pdf) under unknown resolution and data association events. Not all details of the calculation will be given—for that we refer to the paper [7]—but the main steps will be discussed.

The resolution events are incorporated in the calculation of the posterior density as a marginalization over all possible graphs \mathcal{V} ,

$$p(\mathbf{x}_k|\mathbf{Z}^k) = \sum_{\mathcal{V}} p(\mathbf{x}_k, \mathcal{V}|\mathbf{Z}^k), \quad (17)$$

where several graphs can represent the same resolution event, for example as in Fig. 2. To be able to calculate the posterior density, we can rewrite it using Bayes' rule,

$$p(\mathbf{x}_k|\mathbf{Z}^k) = \sum_{\mathcal{V}} \frac{p(\mathbf{Z}_k|\mathcal{V}, \mathbf{x}_k) \Pr\{\mathcal{V}|\mathbf{x}_k\} p(\mathbf{x}_k|\mathbf{Z}^{k-1})}{p(\mathbf{Z}_k|\mathbf{Z}^{k-1})}. \quad (18)$$

Therefore, to express the posterior density, we need the expressions for the graph probabilities $\Pr\{\mathcal{V}|\mathbf{x}_k\}$, and for the likelihood $p(\mathbf{Z}_k|\mathcal{V}, \mathbf{x}_k)$ of the graph and the state vector, which was described in Section 3 for a given data association.

On top of resolution uncertainties, we also have uncertainty in the data association. For known data association, the posterior density is given by (18). The pdf under unknown association is consequently found by marginalizing over all possible data association events. Let $\mathcal{D}(\mathcal{V})$ be the set of data association hypotheses for the graph \mathcal{V} . We then obtain an expression for the posterior density under both resolution and data association uncertainties as

$$p(\mathbf{x}_k|\mathbf{Z}^k) \propto \sum_{\mathcal{V}} \Pr\{\mathcal{V}|\mathbf{x}_k\} \sum_{d \in \mathcal{D}(\mathcal{V})} \Pr\{d|\mathcal{V}, \mathbf{x}_k\} \times p(\mathbf{Z}_k|\mathcal{V}, d, \mathbf{x}_k) p(\mathbf{x}_k|\mathbf{Z}^{k-1}). \quad (19)$$

In (19), $p(\mathbf{x}_k|\mathbf{Z}^{k-1})$ is the predicted density of the target states, which we want to update with information from measurements. In the following sections, we first describe the update with the measurement likelihood, then the update with the data association probabilities, and finally the 'negative information' update with resolution model.

4.1 Update with the measurement likelihood

Here, we summarize the update of the predicted target density function using information from measurements and from the measurement model. The description of the origin of the M_k measurements at time index k is governed by the set of data association hypotheses, $\mathcal{D}(\mathcal{V})$. For each hypothesis d in $\mathcal{D}(\mathcal{V})$, we describe the update of the predicted density, i.e., we present an expression for the product $p(\mathbf{Z}_k|\mathcal{V}, d, \mathbf{x}_k) p(\mathbf{x}_k|\mathbf{Z}^{k-1})$ in (19).

The pdf over the received measurements is expressed in (16). When updating the predicted density, all Gaussian densities in (16) have to be considered. However, since the pdf of received measurements is a product of Gaussian densities, the update can be done sequentially, one Gaussian at a time. Just as the measurement pdf, the predicted density is a product of Gaussians (assuming independent target states at time $k-1$, independent target motions, and a Gaussian approximation over the data association events at time $k-1$)

$$p(\mathbf{x}_k|\mathbf{Z}^{k-1}) = \prod_{i=1}^N p(\mathbf{x}_k^{(i)}|\mathbf{Z}^{k-1}). \quad (20)$$

For the single-target detection, the single-target density $p(\mathbf{x}_k^{(i)}|\mathbf{Z}^{k-1})$ is updated with the corresponding single-

target measurement density. For group targets, the product of Gaussians for the group is updated with the group-measurement density. By viewing a single target as a group target with size one, the description of the update with the likelihood function can be held for a generic group of size one or more.

As previously noted, the update of the predicted density can be performed sequentially for each target-generated measurement at a time (under the described assumptions). In the following, we will describe one of those updates. That is, we will describe the update of the predicted state of a group g , with joint state vector \mathbf{x}_k^g , with the associated measurement $\mathbf{z}_k^{(i)}$. The full update of the predicted density is then given by a sequence of group target updates.

Through the data association hypothesis $d \in \mathcal{D}(\mathcal{V})$ it is known which target generated which detection. For a certain pair of group target \mathbf{x}_k^g (possibly of size one) and associated detection $\mathbf{z}_k^{(j)}$, it holds that

$$p(\mathbf{x}_k^g | \mathbf{Z}^{k-1}) = \mathcal{N}(\mathbf{x}_k^g; \mathbf{x}_{k|k-1}^g, \mathbf{P}_{k|k-1}^g) \quad (21)$$

$$p(\mathbf{z}_k^{(j)} | \mathbf{x}_k^g) = \mathcal{N}(\mathbf{z}_k^{(j)}; \mathbf{H}_{n_g} \mathbf{x}_k^g, \mathbf{R}_k^{n_g}), \quad (22)$$

where \mathbf{H}_{n_g} is the measurement model matrix and $\mathbf{R}_k^{n_g}$ is the measurement noise covariance matrix for a group of n_g targets (cf. (7)). The product of the two Gaussian densities can be written as

$$\begin{aligned} & \mathcal{N}(\mathbf{x}_k^g; \mathbf{x}_{k|k-1}^g, \mathbf{P}_{k|k-1}^g) \mathcal{N}(\mathbf{z}_k^{(j)}; \mathbf{H}_{n_g} \mathbf{x}_k^g, \mathbf{R}_k^{n_g}) \\ &= \mathcal{N}(\mathbf{x}_k^g; \mathbf{x}_{k|k}^g, \mathbf{P}_{k|k}^g) \mathcal{N}(\mathbf{z}_k^{(j)}; \hat{\mathbf{z}}_{k|k-1}^g, \mathbf{S}_k^g), \end{aligned} \quad (23)$$

where expressions for $\mathbf{x}_{k|k}^g$, $\mathbf{P}_{k|k}^g$, $\hat{\mathbf{z}}_{k|k-1}^g$ and \mathbf{S}_k^g are found by identifying terms in the standard Kalman filter equations (see [7] for details). As a function of \mathbf{x}_k^g , the product (23) is a weighted Gaussian.

4.2 Update with the data association probability

The second factor to consider in the calculation of the posterior density (19) is the probability of a data association event d , $\Pr\{d | \mathcal{V}, \mathbf{x}_k\}$.

To obtain a general expression for $\Pr\{d | \mathcal{V}, \mathbf{x}_k\}$ in (19), we define P_D^{u, n_g} as the detection probability for an unresolved group of n_g targets. Then, using the definition of the data association vector d in Section 2,

$$\begin{aligned} \Pr\{d | \mathcal{V}, \mathbf{x}_k\} &= P_c(M_c) \frac{(M_k - M_t)!}{M_k!} \\ &\times \prod_{\{j: d(j)=0\}} (1 - P_D^j) \prod_{\{j: d(j)>0\}} P_D^j, \end{aligned} \quad (24)$$

where $\{j : d(j) = 0\}$ is the set of clutter detections, $\{j : d(j) > 0\}$ is the set of target-generated measurements, and $P_c(M_c)$ is the probability of receiving M_c clutter measurements, which is given by the Poisson mass function with parameter λ . Further, the detection probability P_D^j for measurement index j is equal to P_D if the measurement comes

from a single target and P_D^{u, n_g} if it comes from an unresolved group of $n_g > 1$ targets.

4.3 Update with negative information

In the previous sections, we have described the update of the predicted multi-target pdf using the measurement likelihood and the data association probabilities. What remains is the update of the conditional densities with the resolution probabilities, which are a part of the graph probabilities $\Pr\{\mathcal{V} | \mathbf{x}_k\}$ in (19). As seen in (10), the graph probability is a product of probabilities P_u of unresolved pairs, which in turn are described by scaled Gaussian densities, as seen in (12).

Let us for simplicity start with a part of the full $\Pr\{\mathcal{V} | \mathbf{x}_k\}$ expression, i.e., let us consider the resolution probability for a single unresolved group of n_g targets in the graph. The full resolution probability, then, is the product of the resolution probabilities for all groups and the probability of all open inter-group connections in the graph (there are no edges between vertices of different groups). The group that we consider has a joint state vector \mathbf{x}_k^g and generates a group measurement that is used for updating the joint predicted pdf, as described in (23). Let \mathcal{G}^u be the set of unresolved pairs in the group, for which it holds that $\mathcal{G}^u \subset \mathcal{S}_e$ (cf. (10)). Then, according to (12),

$$\Pr\{\mathcal{G}^u | \mathbf{x}_k^g\} = \prod_{\{i,j\} \in \mathcal{G}^u} |2\pi \mathbf{R}_{u, N_{\text{res}}}|^{1/2} \mathcal{N}(0; \Delta^{i,j} \mathbf{x}_k^g, \mathbf{R}_{u, N_{\text{res}}}). \quad (25)$$

To make the update of the group state, we would like the probability of the unresolved pairs to be a function of \mathbf{x}_k^g , and not a function of the vector $\Delta^{i,j} \mathbf{x}_k^g$, as above. By using the Kronecker delta

$$\delta_{k,i} \triangleq \begin{cases} 1 & \text{if } k = i \\ 0 & \text{otherwise,} \end{cases} \quad (26)$$

the vector

$$\pi^{(i,j)}(k) \triangleq \delta_{k,i} - \delta_{k,j}; \quad k = 1, \dots, n_g, \quad (27)$$

and the matrix

$$\mathbf{\Pi}^{(i,j)} = \pi^{(i,j)} \otimes \mathbf{I}_{N_{\text{res}} \times N_{\text{res}}}, \quad (28)$$

we can write the probability that a pair of targets are unresolved as a function of \mathbf{x}_k^g as (cf. (12))

$$\begin{aligned} \Pr\{\mathcal{G}^u | \mathbf{x}_k^g\} &= \prod_{\{i,j\} \in \mathcal{G}^u} \left\{ \mathcal{N}(\mathbf{0}; \mathbf{\Pi}^{(i,j)} \mathbf{x}_k^g, \mathbf{R}_{u, N_{\text{res}}}) \right. \\ &\quad \left. \times |2\pi \mathbf{R}_{u, N_{\text{res}}}|^{1/2} \right\}. \end{aligned} \quad (29)$$

For one pair of targets (i, j) in the group, we can then perform the re-factorization

$$\begin{aligned} & \mathcal{N}(\mathbf{0}; \mathbf{\Pi}^{(i,j)} \mathbf{x}_k^g, \mathbf{R}_{u, N_{\text{res}}}) \mathcal{N}(\mathbf{x}_k^g; \mathbf{x}_{k|k}^g, \mathbf{P}_{k|k}^g) \\ &= \mathcal{N}(\mathbf{0}; \mathbf{\Pi}^{(i,j)} \mathbf{x}_{k|k}^g, \mathbf{S}_k^{g, ij}) \mathcal{N}(\mathbf{x}_k^g; \mathbf{x}_{k|k}^{g, ij}, \mathbf{P}_{k|k}^{g, ij}), \end{aligned} \quad (30)$$

where the post-refactorization parameters, again, are given by the Kalman filter update equations (see [7] for details).

The ‘negative information’ from a missed detection due to resolution limitations is hence incorporated in the filtering framework as a measured ‘0’ of the separation of a target pair. For the next unresolved pair of targets (l, m) , we do a similar re-factorization of the product $\mathcal{N}(0; \Pi^{(l,m)} \mathbf{x}_k^g, \mathbf{R}_{u, N_{\text{res}}}) \mathcal{N}(\mathbf{x}_k^g; \mathbf{x}_{k|k}^{g,ij}, \mathbf{P}_{k|k}^{g,ij})$.

After the update with all unresolved pairs, we need to update the resolved pairs of the group, represented by the set \mathcal{G}^r , for which it holds that $\mathcal{G} = \mathcal{G}^u \cup \mathcal{G}^r$ is the set of all pairwise connections in the group. The resolution event probability, considering only this group, is then given by (cf. (29))

$$\Pr \{ \mathcal{G} | \mathbf{x}_k^g \} = \Pr \{ \mathcal{G}^u | \mathbf{x}_k^g \} \Pr \{ \mathcal{G}^r | \mathbf{x}_k^g \} \quad (31)$$

with

$$\Pr \{ \mathcal{G}^r | \mathbf{x}_k^g \} = \prod_{\{l,m\} \in \mathcal{G}^r} \left\{ 1 - |2\pi \mathbf{R}_{u, N_{\text{res}}}|^{1/2} \times \mathcal{N}(0; \Pi^{(l,m)} \mathbf{x}_k^g, \mathbf{R}_{u, N_{\text{res}}}) \right\}. \quad (32)$$

As seen in (32), the probability of resolution for a pair of targets is one minus the probability that they are unresolved. Performing a negative information update for that pair, similarly to (30), gives two Gaussian components. Thus, each update with a resolved pair of targets doubles the number of Gaussian components.

Alternatively to the sequential update considered here, it is possible to update each product of exponential factors by a single re-factorization in an extended target state space using block matrices of the dimension $D = \dim(\mathbf{x}_k^g) \times |\mathcal{G}^u|$. Details of the sequential and the block matrix update will be presented in a separate publication [9].

We now have discussed the update of a group target state using the part of the graph probability $\Pr \{ \mathcal{V} | \mathbf{x}_k \}$ which includes the pair-wise connections within a group. This defines a procedure to update several separated groups. What remains to handle are the connections between groups, i.e., the open links (due to resolvability) between vertices of different clusters, which are part of the graph probability (represented as a $(1 - P_u)$ factor, cf. (10)). The treatment of inter-connections can be handled in the same way as the update of a group state, if a new state vector is formed, which contains the state vectors under consideration. Then, update in the same way as presented can be performed. However, the number of updates increases dramatically, since all combinations of two, three, four and more combinations of pairwise open edges have to be considered. But, when target groups are well separated, the P_u factors are very small, and in those cases the updates including that pair of targets can be neglected. In practice, this is anticipated to occur in most situations. Then, each group can be handled separately, without interactions.

5 JPDA filter incorporation

As described in Section 4, the calculation of the posterior density under unknown data association and resolution

events includes the sum over a large number of multiplications of Gaussian densities and scaling factors. The density after update (assuming it was Gaussian before the update) can be expressed as

$$p(\mathbf{x}_k | \mathbf{Z}^k) = \sum_{\mathcal{V}} \sum_{u \in \mathcal{U}(\mathcal{V})} \sum_{d \in \mathcal{D}(\mathcal{V})} c^{\mathcal{V},u,d} \mathcal{N}(\mathbf{x}_k; \mathbf{x}_{k|k}^{\mathcal{V},u,d}, \mathbf{P}_{k|k}^{\mathcal{V},u,d}), \quad (33)$$

where the scaling $c^{\mathcal{V},u,d}$ depends on the data association probability $\Pr\{d | \mathcal{V}, \mathbf{x}_k\}$ (cf. (24)), the graph and state vector likelihood $p(\mathbf{Z}_k | \mathcal{V}, d, \mathbf{x}_k)$ (cf. (16)) and the graph probability $\Pr\{\mathcal{V} | \mathbf{x}_k\}$ (cf. (10)). The sum over $u \in \mathcal{U}$ comes from the negative-information update, where each update with a resolved pair of targets doubles the number of Gaussian components (see the discussion after (32)). The scaling is normalized such that the sum over $c^{\mathcal{V},u,d}$ is one. The expected value $\mathbf{x}_{k|k}^{\mathcal{V},u,d}$ and covariance matrix $\mathbf{P}_{k|k}^{\mathcal{V},u,d}$ of the joint state vector under a resolution and data association hypothesis is found by a sequence of measurement updates (one for each group in the graph) and negative information updates (one for each pairwise connection in the graph).

The standard JPDA filter makes a Gaussian approximation of a Gaussian mixture at each time step. Similarly, a JPDA filter with a resolution model should make a Gaussian approximation of the mixture in (33). We present two alternatives of making the approximation:

1. Perform a moment matching over the full Gaussian mixture in (33).
2. First perform a moment matching over the data association hypotheses for a given graph, and then make a second moment matching over the set of graphs, after negative information update.

While the first alternative is more exact, the second option requires much less computations.

By A_1 we denote the first approximation, and by A_2 the other one. Then, under the two alternatives, we make the approximations

$$p(\mathbf{x}_k | \mathbf{Z}^k) \approx \mathcal{N}(\mathbf{x}_k; \mathbf{x}_{k|k}^{A_i}, \mathbf{P}_{k|k}^{A_i}), \quad (34)$$

where

$$\mathbf{x}_{k|k}^{A_1} = \sum_{\mathcal{V}} \sum_{u \in \mathcal{U}(\mathcal{V})} \sum_{d \in \mathcal{D}(\mathcal{V})} c^{\mathcal{V},u,d} \mathbf{x}_{k|k}^{\mathcal{V},u,d} \quad (35)$$

$$\mathbf{P}_{k|k}^{A_i} = \sum_{\mathcal{V}} \sum_{u \in \mathcal{U}(\mathcal{V})} \sum_{d \in \mathcal{D}(\mathcal{V})} c^{\mathcal{V},u,d} \left\{ \mathbf{P}_{k|k}^{\mathcal{V},u,d} + (\mathbf{x}_{k|k}^{\mathcal{V},u,d} - \mathbf{x}_{k|k}^{A_1}) (\mathbf{x}_{k|k}^{\mathcal{V},u,d} - \mathbf{x}_{k|k}^{A_1})^T \right\} \quad (36)$$

for the first alternative, and

$$\mathbf{x}_{k|k}^{A_2} = \sum_{\mathcal{V}} \sum_{u \in \mathcal{U}} c^{\mathcal{V},u} \bar{\mathbf{x}}_{k|k}^{\mathcal{V},u} \quad (37)$$

$$\mathbf{P}_{k|k}^{A_2} = \sum_{\mathcal{V}} \sum_{u \in \mathcal{U}} c^{\mathcal{V},u} \left\{ \mathbf{P}_{k|k}^{\mathcal{V},u} + (\bar{\mathbf{x}}_{k|k}^{\mathcal{V},u} - \mathbf{x}_{k|k}^{A_2}) (\bar{\mathbf{x}}_{k|k}^{\mathcal{V},u} - \mathbf{x}_{k|k}^{A_2})^T \right\} \quad (38)$$

for the second alternative. The state vector $\bar{\mathbf{x}}_{k|k}^{\mathcal{V},d}$ and covariance matrix $\mathbf{P}_{k|k}^{\mathcal{V},u}$ are given by first making a moment matching over the data association hypotheses for a given graph \mathcal{V} ,

$$\sum_{d \in \mathcal{D}(\mathcal{V})} c^{\mathcal{V},d} \mathbf{x}_{k|k}^{\mathcal{V},d} \approx \mathcal{N}(\mathbf{x}_k; \bar{\mathbf{x}}_{k|k}^{\mathcal{V},d}, \mathbf{P}_{k|k}^{\mathcal{V},u}), \quad (39)$$

where $c^{\mathcal{V},d}$ is given by the data association probability and the measurement likelihood, and then updating the resulting Gaussian density with negative information using the resolution model. Similarly, $c^{\mathcal{V},u}$ is given by the negative-information update, using $\Pr\{\mathcal{V}|\mathbf{x}_k\}$ and $\mathcal{N}(\mathbf{x}_k; \bar{\mathbf{x}}_{k|k}^{\mathcal{V},d}, \mathbf{P}_{k|k}^{\mathcal{V},u})$ (see (29), (30)).

6 Simulation Results

In this section we study a simple simulated tracking example with two targets. In the scenario, the targets are first approaching each other, then move in parallel, and finally separate. Due to limited sensor resolution, the two targets are not always resolved. Since the targets are closely spaced, there is a big risk of track coalescence in the ordinary JPDA filter. Example output from the JPDA filter without and with resolution model incorporation can be found in Figs. 3 and 4, respectively. As seen in the figures, the tracks from the JPDA filter with resolution model are more separated, due to a better description of the received measurements. The targets move with a constant speed of 5 m/s, and the true separation of the targets is 40 m in the parallel section.

At time intervals, T , of one second a sensor measures the x and y position of the targets. The measurement noise is governed by (cf. (7))

$$\mathbf{R}^1 = \begin{bmatrix} 16 & 0 \\ 0 & 16 \end{bmatrix}, \quad \mathbf{R}^2 = \begin{bmatrix} 64 & 0 \\ 0 & 64 \end{bmatrix} \quad (40)$$

for single and group target measurements, respectively.

For prediction of future states, a (nearly) constant velocity model is used, with system matrix $\mathbf{F} = [\tilde{\mathbf{F}} \quad \tilde{\mathbf{F}}]$ (cf. (9)),

$$\tilde{\mathbf{F}} = \begin{bmatrix} 1 & 0 & T & 0 \\ 0 & 1 & 0 & T \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad (41)$$

and process noise covariance matrix $\mathbf{Q} = [\tilde{\mathbf{Q}} \quad \tilde{\mathbf{Q}}]$, where

$$\tilde{\mathbf{Q}} = q_0 \begin{bmatrix} T^3/3 & 0 & T^2/2 & 0 \\ 0 & T^3/3 & 0 & T^2/2 \\ T^2/2 & 0 & T & 0 \\ 0 & T^2/2 & 0 & T \end{bmatrix}, \quad (42)$$

and $q_0 = 0.6$.

The sensor resolution model is used for generation of measurements. The resolution capabilities of the sensor are described by the parameters α_x and α_y , see (13). In this scenario, both resolution parameters are set to 40 m. The

probability of detection is 0.95 and the average number of clutter measurements is 5 per scan.

For evaluation of tracking performance, two measures are used. The first one is the Mean Optimal Subpattern Assignment (MOSPA) measure [10], which is a measure that disregards target identity and thus only considers the estimation of where there are targets. The second measure is the Root Mean Square Error (RMSE) of the position error.

In Fig. 5, the MOSPA performance, averaged over 500 Monte Carlo runs, is shown for the JPDA filter with and without resolution model, and for the case of perfect resolution, i.e., when the targets are always resolved. The results clearly show that the JPDA filter with resolution model performs better than the filter without resolution model.

The performance in RMSE sense, averaged over 500 Monte Carlo runs, is shown in Fig. 6. Obviously, also the RMSE performance is improved when the resolution model is incorporated in the JPDA filter. The increased RMSE of the JPDA filter in the end of the scenario is due to the fact that there is a larger probability of track switch for that filter, compared to the filter with resolution model.

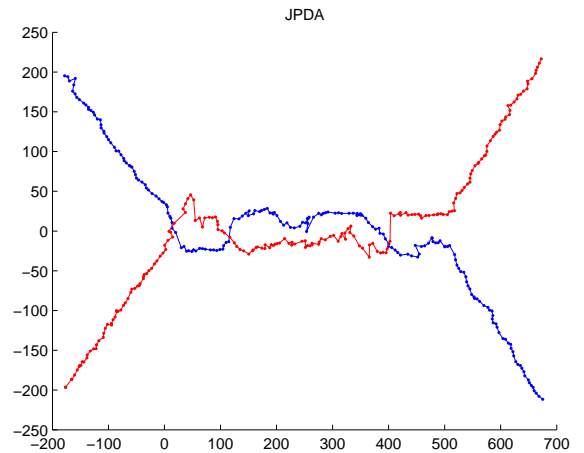


Figure 3: Typical tracking result for the JPDAF without resolution model.

7 Summary and Outlook

We have proposed a sensor resolution model for arbitrary target numbers, which is an extension of the two-target model by Koch and van Keuk [3]. The resolution model leads to additional data association possibilities and to a multi-target likelihood function that takes missed detections due to merged measurements into account. The corresponding posterior density function is derived explicitly. As the filter update, in general, is infeasible, the extensive Gaussian mixture has been approximated by a JPDA filter. The application to a simple target tracking scenario shows a significantly improved tracking performance if the sensor resolution model is used. While these first results are encouraging, more simulations are needed to confirm the practicability of

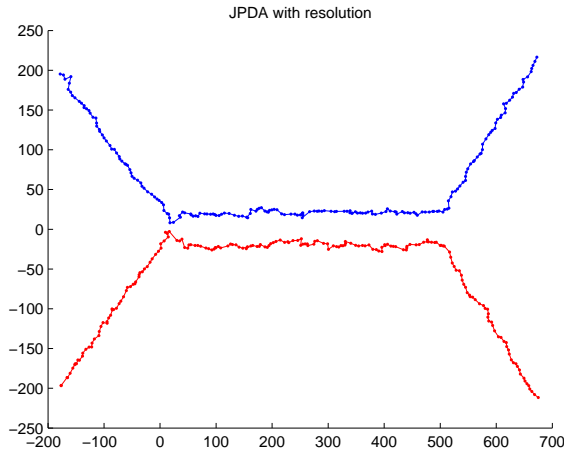


Figure 4: Typical tracking result for the JPDAF with resolution model.

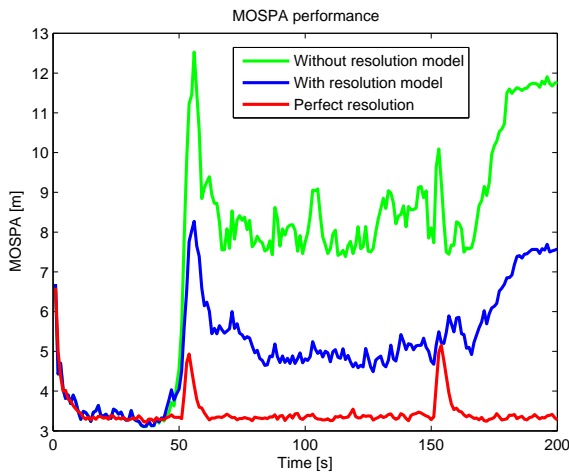


Figure 5: MOSPA performance of the JPDAF without resolution model (top), with resolution model (middle) in comparison to the case of perfect sensor resolution (bottom).

the presented approach. In particular, the stability against resolution model mismatch needs to be investigated. Furthermore, the resolution model and the corresponding likelihood function can also be integrated into more sophisticated tracking filters such as the Multiple Hypothesis Tracker or the Probability Hypothesis Density Filter.

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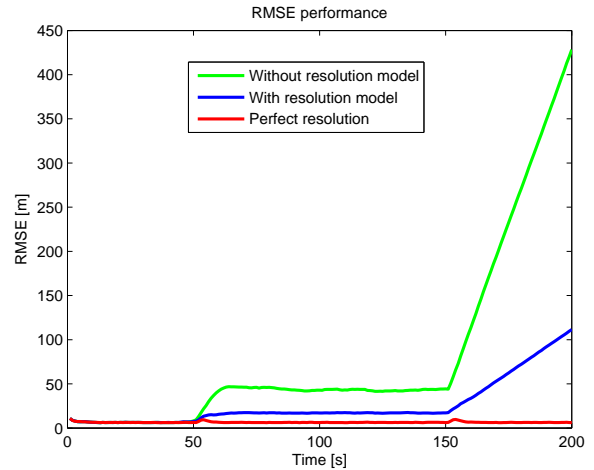


Figure 6: RMSE performance of the JPDAF without resolution model (top), with resolution model (middle) in comparison to the case of perfect sensor resolution (bottom).

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